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# Toward exact number: Young children use one-to-one correspondence to measure set identity but not numerical equality



Véronique Izard <sup>a,b,c,\*</sup>, Arlette Streri <sup>a,b</sup>, Elizabeth S. Spelke <sup>c</sup>

<sup>a</sup> Laboratoire Psychologie de la Perception, Université Paris Descartes, Sorbonne Paris Cité, 45 rue des Saints-Pères, 75006 Paris, France

<sup>b</sup> CNRS UMR 8158, 45 rue des Saints-Pères, 75006 Paris, France

<sup>c</sup> Department of Psychology, Harvard University, 33 Kirkland St., Cambridge, MA 02138, USA

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## ABSTRACT

Exact integer concepts are fundamental to a wide array of human activities, but their origins are obscure. Some have proposed that children are endowed with a system of natural number concepts, whereas others have argued that children construct these concepts by mastering verbal counting or other numeric symbols. This debate remains unresolved, because it is difficult to test children's mastery of the logic of integer concepts without using symbols to enumerate large sets, and the symbols themselves could be a source of difficulty for children. Here, we introduce a new method, focusing on large quantities and avoiding the use of words or other symbols for numbers, to study children's understanding of an essential property underlying integer concepts: the relation of exact numerical equality. Children aged 32–36 months, who possessed no symbols for exact numbers beyond 4, were given one-to-one correspondence cues to help them track a set of puppets, and their enumeration of the set was assessed by a non-verbal manual search task. Children used one-to-one correspondence relations to reconstruct exact quantities in sets of 5 or 6 objects, as long as the elements forming the sets remained the same individuals. In contrast, they failed to track exact quantities when one element was added, removed, or substituted for another. These results suggest an alternative to both nativist and symbol-based constructivist theories of the development of natural number con-

\* Corresponding author at: Laboratoire Psychologie de la Perception, Université Paris Descartes, Sorbonne Paris Cité, 45 rue des Saints-Pères, 75006 Paris, France. Fax: +33 (0)1 42 86 21 97.

E-mail address: [veronique.izard@polytechnique.org](mailto:veronique.izard@polytechnique.org) (V. Izard).

cepts: Before learning symbols for exact numbers, children have a partial understanding of the properties of exact numbers.

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## 1. Introduction

Number is one of the core competences of the human mind (Carey, 2009; Dehaene, 1997; Dehaene & Brannon, 2011). From birth, human infants discriminate between sets on the basis of number (Feigenson, Dehaene, & Spelke, 2004; Izard, Sann, Spelke, & Streri, 2009; Xu & Spelke, 2000), and by the first few months of life, they can perform simple numerical additions, subtractions, and comparisons (Brannon & Van de Walle, 2001; McCrink & Wynn, 2004, 2007; Wynn, 1992). To account for these competences, current theories grant infants two core systems capable of encoding numerical information (Carey, 2009; Feigenson et al., 2004; Hyde, 2011). These two systems are associated with infants' numerical capacities with large and small sets, respectively. First, infants can represent, compare, and perform arithmetic operations on large approximate numerosities. Second, infants can track small sets of up to 3 or 4 objects, and through these attentional abilities, they can solve simple arithmetic tasks involving small exact numbers of objects.

Yet, infants' sensitivity to number shows striking limitations when compared to the power of the simplest mathematical numbers: the integers, or "natural numbers." In the large number range (beyond 3 items), infants' discrimination of numerosities is approximate and follows Weber's law: numerosities can be discriminated only if they differ by a minimal ratio (Xu, Spelke, & Goddard, 2005). The same imprecise representations are found in young children and even in educated adults, when they are prevented from counting (Halberda & Feigenson, 2008; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Piazza et al., 2010). Because of this limitation, numerosity perception fails to capture two essential properties that are central to formalizations of the integers: the relation of exact numerical equality and the successor function (Izard, Pica, Spelke, & Dehaene, 2008; Leslie, Gelman, & Gallistel, 2008). The relation of exact numerical equality grounds integers in set-theoretic constructions: two sets are equinumerous if and only if their elements can be placed in perfect one-to-one correspondence (this is Hume's principle). The successor function, on the other hand, is the initial intuition underpinning the Peano–Dedekind axioms: here the integers are generated by successive additions of one, i.e., by the iteration of a successor operation.

Theories diverge with regards to the origins of the concept of exact number in children's development. Some have proposed that exact number is innate, either because the properties of exact number are built into the system of analog mental magnitude (Gelman & Gallistel, 1986), or because there is a separate system giving children an understanding of exact equality and/or of the successor function (Butterworth, 2010; Hauser, Chomsky, & Fitch, 2002; Leslie et al., 2008; Rips, Bloomfield, & Asmuth, 2008). For example, Leslie et al. proposed that children have an innate representation of the exact quantity ONE that can be used iteratively to generate representations of exact numbers. In the same vein, Frank, Everett, Fedorenko, and Gibson (2008) and Frank, Fedorenko, Lai, Saxe, and Gibson (2011) proposed that humans can represent one-to-one correspondence non-symbolically and know intuitively that perfect one-to-one correspondence entails exact numerical equality. Other theorists have proposed that the concept of exact number is constructed, and that symbols such as tally marks, numerical expressions in natural language, abacus configurations, or other symbolic devices play a crucial role in this process (Bialystok & Codd, 1997; Carey, 2009; Cooper, 1984; Fuson, 1988; Klahr & Wallace, 1976; Mix, Huttenlocher, & Levine, 2002; Piantadosi, Tenenbaum, & Goodman, 2012; Schaeffer, Eggleston, & Scott, 1964; Spelke, 2003). For example, Carey (2009) proposed that children construct the natural numbers by (1) learning the ordered list of count words as a set of uninterpreted symbols, then (2) learning the exact meanings of the first three or four count words, mapping the words to representations of 1–4 objects that are attended in parallel, and finally (3) constructing an

analogy between the serial ordering of the list of number words and the numerical ordering of the arrays of objects.

To address the debate on the origins of exact number concepts, here we focused on one of the fundamental properties of the integers: the relation of exact numerical equality between sets. We asked whether children understand this relation before they master linguistic or other symbols for exact numbers.

### 1.1. The relation of exact numerical equality

As we mentioned above, the set-theoretic definition of exact numbers relies on Hume's principle: two sets are equal in number if and only if they can be placed in perfect one-to-one correspondence. This definition entails a list of characteristic principles of the relation of numerical equality, derived by analyzing the impact of different types of transformations on two initially equal sets. Following a strategy first put forward by Gelman and Gallistel in their study of counting (Gelman, 1972a; Gelman & Gallistel, 1986), we first articulate three principles and then use them to assess children's understanding of the relation of exact numerical equality. Crucially, our tests allow for the possibility that children may understand some, but not all, of these principles.

- (1) *The Identity principle*: If two sets are equal in number, they remain equal over transformations that do not affect the identity of any member of either set, such as changes in the spatial positions of one set's members.
- (2) *The Addition/Subtraction principle*: If two sets are equal in number, an addition or subtraction transformation applied to one of the sets disrupts the equality, even for minimal transformations of one item.
- (3) *The Substitution principle*: Numerical equality is maintained over a different kind of transformation to one set: the substitution of one element by another item.

In the rest of this section, we show that each of these principles is a necessary constituent of the relation of exact equality, and therefore a child could not be granted knowledge of exact equality if he/she did not subscribe to all three principles. To do so, we show that waiving one or the other of these principles still leads to coherent relations between sets, but not necessarily to the relation of exact numerical equality. We also establish the relevance of our principles to cognitive development, as waiving one or more of our three principles enables us to capture the different hypotheses put forward in the literature on children's number concepts.

Let us assume first that children judge numerical equality based on perceptual similarity between numerosities – in other words, that they are limited to a relation of approximate numerical equality. A relation of approximate equality follows the *Identity* and *Substitution* principles, but not necessarily the *Addition/Subtraction* principle. Under approximate equality, in accordance with the *Identity* and *Substitution* principles, two sets remain approximately equal in number after the elements of the sets have been displaced, or after one element has been substituted for another item. However, contrary to the *Addition/Subtraction* principle, a child may judge a set to retain the same approximate number of elements after an addition or subtraction, provided that the ratio difference produced by the transformation lies below his or her threshold for numerical discrimination. Understanding the *Addition/Subtraction* principle is therefore diagnostic of children's reasoning about exact as opposed to approximate quantities.

Alternatively, early research by Piaget (1965) suggested that young children do not take the relation "same number" to follow the *Identity* principle, since children judge two matching lines of objects to become unequal in number after one of the arrays is spread out. Piaget's interpretation was later contested, by appealing either to the pragmatics of the tasks by which numerical judgments were elicited (Gelman, 1972b; Markman, 1979; McGarrigle & Donaldson, 1974; Siegel, 1978) or to the demands imposed on children's executive resources (Borst, Poirel, Pineau, Cassotti, & Houdé, 2012). Nevertheless, Piaget's interpretation of the child's concept of number can easily be captured through the principles put forward above, as a failure to understand *Identity*. The *Identity* principle is thus

diagnostic in this case, because children might still judge the *Addition/Subtraction* and *Substitution* principles to hold.

Finally, one could define yet another type of relation between sets, by waiving only the *Substitution* principle. Without this principle, two sets may be judged unequal just because they are formed of different individuals, because *Identity* and *Addition/Subtraction* alone do not suffice to construct two sets that are different, yet equal. Again, negating the *Substitution* principle would still be compatible with both the *Identity* and *Addition/Subtraction* principles. Consider, for example, a set specified by the identity of its members, such as the set of members of a family. This set changes with the replacement of a family member by an unrelated individual (contrary to the *Substitution* principle) but is maintained over movements of its individual members (in accord with the *Identity* principle) and grows with the addition of new members (in accord with the *Addition/Subtraction* principle).

In summary, the principles of *Identity*, *Addition/Subtraction*, and *Substitution* jointly serve to characterize the formal relation of exact numerical equality, since different relations can be defined by waiving one or another principle. Past research indicates that children sometimes provide judgments that accord with these deviant relations. In the present research, we investigated whether children can represent the property of sets that accords to all three principles: exact numerical equality.

## 1.2. Early understanding of exact quantities

### 1.2.1. Small quantities

In stark contrast to [Piaget's \(1965\)](#) theory, Gelman demonstrated that, when tested in the small number range, even very young children are sensitive to exact differences in numerosity ([Gelman, 1972b, 2006](#); [Gelman & Gallistel, 1986](#)). For example, when given the instruction that one of two plates containing respectively 2 and 3 objects was 'the winner' (thus avoiding any reference to number words), children under 3 years could recognize the target numerosity after the objects were displaced, detected a change in number after the experimenter had surreptitiously added or removed an object from a plate, and even offered a solution to undo the change. These results were later extended in research with preverbal infants, who also proved able to detect a contrast between 2 and 3 objects ([Feigenson et al., 2004](#); [Féron, Gentaz, & Streri, 2006](#); [Kobayashi, Hiraki, & Hasegawa, 2005](#); [Kobayashi, Hiraki, Mugitani, & Hasegawa, 2004](#); [Wynn, 1992](#); see [Bisazza, Piffer, Serena, & Agrillo, 2010](#); [Rugani, Fontanari, Simoni, Regolin, & Vallortigara, 2009](#), for a demonstration of the same abilities in newly hatched fish and chicks).

Nevertheless, young children's sensitivity to exact small numerosities can be explained in three different ways. First, children may represent sets of 1, 2, or 3 objects as having distinct integer values, as Gelman and Gallistel proposed ([Gallistel & Gelman, 1992](#); [Gelman & Gallistel, 1986](#)). Second, children may represent these sets as having distinct approximate numerical magnitudes, discriminating between them exactly only because these small numbers differ from one another by large ratios ([Dehaene & Changeux, 1993](#); [van Oeffelen & Vos, 1982](#)). Third, children may represent these sets by the mechanism of parallel tracking, whereby exact small numerosities are represented in a separate format, through object files serving to index 1–3 individual objects ([Feigenson et al., 2004](#); [Hyde, 2011](#); [Simon, 1997](#)). In the latter case, children may represent the extension of a set (i.e., that the set is composed of objects A, B, and C) without representing its cardinal value. The latter two possibilities grant the youngest children an ability to process small numerosities in an exact fashion, but without postulating that they do so by drawing on the integer concepts used by adults. These three accounts can only be distinguished by research investigating whether the above abilities extend to the large number range.

Unfortunately, the studies developed with small numbers cannot easily be extended to larger numbers, because perception is approximate in this range ([Gelman, 2006](#)). Yet, the fact that perception is approximate does not preclude that children, like adults, may recognize that the numerosity of a set is changed when one element is added to or removed from the set, and that it is preserved when both of these operations are performed. In order to test for this possibility, previous research has turned to children's understanding of number words, guided by the assumption that the way children interpret numerical symbols may reveal what kind of numerical concepts they spontaneously entertain ([Condry & Spelke, 2008](#); [Fuson, 1988](#); [Huang, Spelke, & Snedeker, 2010](#); [Le Corre & Carey, 2007](#); [Le Corre, Van](#)

de Walle, Brannon, & Carey, 2006; Lipton & Spelke, 2006; Mix et al., 2002; Sarnecka & Carey, 2008; Sarnecka & Gelman, 2004). Therefore, we now turn to studies of children's number word learning.

### 1.2.2. Children learning numerals

By the age of 5 years, children clearly recognize that the principles of exact numerical equality govern the usage of number words (Lipton & Spelke, 2006). To demonstrate this ability, Lipton and Spelke presented 5-year-old children with a box full of objects and used a numerical expression to inform the children of the number of objects contained in this box (e.g., “this box has eighty-seven marbles”). Next, the experimenter applied a transformation to the set by subtracting one object, by subtracting half of the objects, or simply by shaking the box. The children rightly judged that the original number word ceased to apply after a subtraction, even of just one item, but not after the box had been shaken. Moreover, they returned to the original number word after the transformation was reversed by the addition of one object, even when the object taken from the original set was replaced by a different object. Crucially, the children showed this pattern of responses not only with number words to which they could count, but also with words beyond their counting range.

Nevertheless, 5-year-old children have had years of exposure to number words. To address the debate on the origins of integer concepts, researchers have thus turned to younger children near the onset of number word learning. Do these younger children understand that number words refer to precise numerical quantities as soon as they recognize that these words refer to numbers?

Learning the verbal numerals starts around the age of 2 and progresses slowly (Fuson, 1988; Wynn, 1990). Children between the ages of 2 and 3½ typically can recite number words in order up to “ten”, but map only a subset of these words (usually only the first three number words or fewer) to exact cardinal values. For these children (hereafter, “subset-knowers”), number word knowledge is often assessed by asking them to produce sets of verbally specified numbers (hereafter, the “Give-N” task; Wynn, 1990). Among subset-knowers, some children succeed only for “one” (“one-knowers”) and produce sets of variable numerosity (but never sets containing just one object) for all other number words; other children show this pattern of understanding for “two” or even “three” and “four”, but produce larger sets of variable numerosity when asked for larger numbers.

Children at this stage are thought to lack an understanding of the cardinal principle, i.e., the fact that the last word uttered in a count refers to the number of objects in the set that was counted. In contrast, children succeeding at the give-N task are usually referred to as “Cardinal Principle Knowers” (hereafter, CP-Knowers). Becoming a CP-Knower has been thought to mark a crucial induction where children construct a new concept of exact number (Carey, 2009; Piantadosi et al., 2012; although see Davidson, Eng, & Barner, 2012). Thus, to address the debate on the origins of exact numbers, in the rest of this paper we focus on the number concepts of children who have not yet mastered counting: subset-knowers.

Do subset-knowers understand that number words refer to precise quantities, defined in terms of exact equality? In the small number range, by definition, subset-knowers apply their known number words to exact quantities, as do adults. To be classified as a “two-knower”, for example, a child must systematically give exactly one and two objects when asked for one and two objects respectively, and he/she must not give one or two objects when asked for other numbers. In line with this competence, for quantities within the range of their known number words, children's interpretation of number words accords with the relation of exact numerical equality (Condry & Spelke, 2008): children choose a different number word after a transformation that affects one-to-one correspondence (such as addition), but not after a transformation that does not affect the set (such as rearrangement). Nevertheless, these abilities are open to the same three interpretations as is children's performance in Gelman's “winner” task (Gelman, 1972a, 2006; Gelman & Gallistel, 1986): Known number words may designate exact cardinal values; they may designate approximate numerosities (and yield exact responding because of the large ratio differences between sets of 1, 2, and 3); or the meaning of these words may be defined through representations constructed in terms of parallel object tracking, a mechanism that is not available for larger numerosities. Studies of subset-knowers' application of larger number words are needed to determine whether subset-knowers interpret exact numerals in terms of exact numbers.

In contrast to their performance with words for small numbers, subset-knowers do not consistently apply words for larger numbers to precise quantities, even for words that they use when they engage

in the counting routine. Results are mixed across studies (Brooks, Audet, & Barner, 2012; Condry & Spelke, 2008; Sarnecka & Gelman, 2004), and different interpretations have been proposed for these discrepant results: children's responses may either reflect limits to their conceptual competence, or variations of their strategic performance (Brooks et al., 2012). We will return to this debate in the General Discussion; at this point, it suffices to note that subset-knowers do not consistently generalize number words according to exact number. Therefore, the study of children's interpretation of number words does not yet reveal whether subset-knowers use exact numerical equality in applying number words.

Even if subset-knowers do not interpret number words as referring to precise quantities, however, this failure need not imply that they fail to understand exact numerical equality in non-linguistic contexts. Children could very well favor alternative interpretations for number words, even if they have a concept of exact numerical equality (see Huang et al., 2010 for evidence that when subset-knowers are trained on the number words beyond their knowledge level, they sometimes interpret these new number words in terms of approximate quantity). Indeed, an interpretation of number words in terms of approximate quantity might receive more support from experience than an interpretation in terms of exact quantity. When children hear number words, they usually do not have the means to register the exact number of objects presented. According to some theories, moreover, number words have inexact meanings even for adults, who use pragmatic inferences to restrict number word reference in some contexts. These meanings may extend to children, whose usage of number words is further limited by the demands of making the appropriate pragmatic inferences (Barner & Bachrach, 2009).

In summary, studies of children's number word learning and interpretation provide suggestive, but not conclusive, evidence bearing on young children's numerical concepts. Therefore, in our search for the origins of the concept of exact number, we constructed a task testing children's knowledge of the relation of exact numerical equality without calling on number words. In this task, we provided subset-knowers with one-to-one correspondence cues to make exact discriminations between quantities available to perception, and we tested children's ability to use these cues to give judgments on exact quantities. Across experiments, we asked whether subset-knowers would interpret one-to-one correspondence mappings in accordance with the three principles of numerical equality described above: one-to-one mappings between two sets are preserved as long as the elements in the two sets remain identical, they change when a single item is added to or taken from one of the sets, and they remain constant over a substitution, within one set, of one item for another.

### 1.3. Overview of the experiments

All the children included in the studies were less than 3 years of age and failed to understand the exact meaning of number words beyond *four*, as assessed by a give-N task. In five experiments, participants were presented with a set of finger puppets placed in one-to-one correspondence with the branches of a toy tree, which, in most conditions, made a difference of one puppet easily detectable. In each trial, the puppets were taken from the tree and placed in an opaque box, while the experimenter narrated and acted out a story that sometimes resulted in a transformation of the set (addition, subtraction, substitution, or identity-preserving transformation). Children's representation of the set of puppets in the box was assessed by allowing children to retrieve either all the puppets, or all but one puppet, and then measuring the time children spent searching in the box for more puppets.

## 2. General methods

### 2.1. Participants

Ninety-three children (53 females) aged 32:08 to 35:26 (months:days) participated in the experiments. An additional 33 children were excluded because of video equipment failure (3), error in the procedure (2), because the children refused to participate or follow instructions (13), they were not native speakers of English (3), they were found to succeed at the give-N task (see procedures and data analyses below) (11), or because they could not be classified as either a subset-knower or a CP-knower



(1). All the participants were recruited by mail, email, or phone based on commercially available lists of contacts for the greater Boston area. Children were mostly Caucasian from a middle-class background, although some African- and Asian-American children were tested as well. The study was approved by the Institutional Review Board (IRB) of Harvard University. Written consent was obtained from one or both parents, and the children gave oral consent.

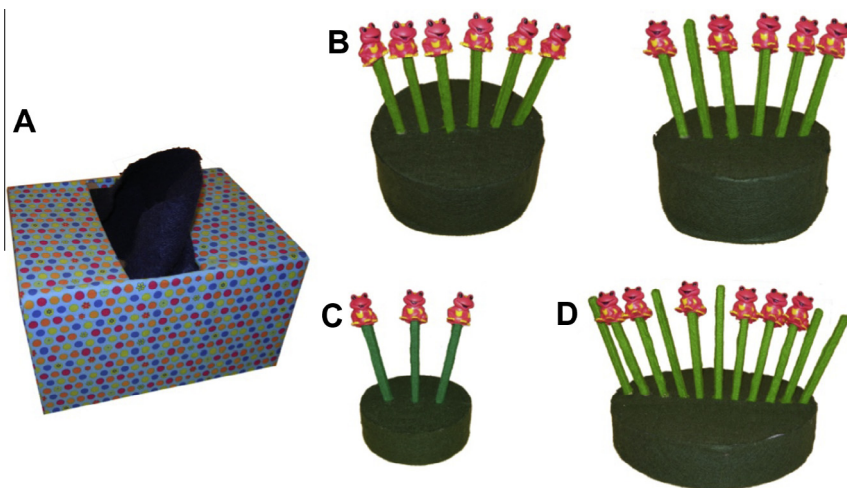
Participants could only be included in the analyses if they had one valid trial in each of two conditions: box expected to be empty and box expected to contain one puppet (see below for the trial inclusion criteria). These criteria resulted in final samples of 12–36 subset-knowers per experiment; the groups are described in the Method section of each experiment. Detailed information on the number of subjects and trials excluded in each experiment are provided in the [Appendix in Table A.1](#).

## 2.2. Displays

Displays were sets of identical animal finger puppets made of rubber. Different animals were used across trials to maintain interest. The animals could be placed on the branches of a “tree”, a custom-made device with sticks protruding in a line ([Fig. 1](#)). An opaque box covered with colorful fabric served as the hiding box. The box had an opening on the top, which could be covered by a piece of felt to fully hide its contents.

## 2.3. Procedure

Children were tested in a quiet laboratory room with their caregiver seated behind them. All experiments started with the same three training trials. In the first training trial, two animal puppets, perceptibly different from each other, were placed on two branches of a tree with 6 branches. The children were then told that night was coming, and the puppets wanted to go sleep in their box. After the puppets were placed in the box there was a short delay, and then the experimenter and the child proceeded to wake up the puppets: the experimenter knocked on the box, searched and got the first puppet, placed it on the tree, and then encouraged the child to get the other puppet. The second training trial was identical to the first, but with three puppets (still different from each other), and the child was expected to find two of them by her/himself. When all three puppets were placed back on the tree, the experimenter asked, “Now do we have all the puppets?” Most children answered



**Fig. 1.** Displays used in the experiments. (A) Searching box. (B) Example of sets of 6 and 5 puppets placed on a ‘tree’ with 6 branches. (C) Example of a set of 3 puppets placed on a tree with 3 branches in Experiment 3. (D) Example of a set of 6 puppets placed on a tree with 11 branches in Experiment 5.



affirmatively; if they did not, they were encouraged to search again in the box, and since they could not find anything, the experimenter stated that all puppets were present.

The third and final training trial was intended to emphasize that the branches could be used as cues for tracking the puppets. The trial started with 5 (perceptibly different) puppets placed on 5 of the 6 branches. Once the puppets were in place, the experimenter pointed to the empty branch, and explained that since no puppet was sitting on that branch, a flower would be placed on it. The flower was attached to the base of the branch with a magnet. After that, the trial unfolded as the previous ones: the puppets first went to sleep in the box, and then they came back to the tree after a short delay. The experimenter helped the child to find the first three puppets and place them on the tree. If the child placed one puppet on the branch with the flower, the experimenter explained that nobody should be placed on that branch because of the flower. If the child insisted on placing a puppet on that particular branch, the experimenter moved the flower. If a second attempt was made to place another puppet on the branch with the flower, the experimenter did not comment and let the child place the puppet there. After three puppets were retrieved, the child was handed the box, with the request, “Can you look for the rest?” If the child stopped searching at some point, the experimenter asked, “Now do we have all the puppets?” If the child answered positively, and a puppet was missing, the experimenter pointed to an empty branch (without the flower) and said, “But nobody is sitting here, there must be another puppet in the box”. If all puppets were already placed back on the branches, the experimenter pointed to the branch with the flower (moving the flower to the empty branch if needed) and said, “Here we have a flower, so nobody should be seating on that branch. We have all the puppets!”

Following the training procedure, each child was given four experimental trials (either four trials in the same experiment or two blocks of two trials in different experiments). In contrast to the training procedure, at test sets were all made of identical puppets. The number of puppets and branches on the tree varied across experiments and will be described below. Nevertheless, in each experiment the child received at least two trials that differed from each other only in terms of one puppet, thus allowing us to record the impact of this minimal difference on the searching behavior of the child.

At the beginning of a trial, all the puppets were placed on the branches of the tree. Most of the time (except in Experiment 5), the starting situation involved either the same number of puppets and branches, or one fewer puppets than branches. To make sure that the children encoded the starting situation, the experimenter gave a verbal description: “We have puppets everywhere on that tree!” or, pointing to the empty branch: “Nobody is sitting here. But it is ok, it is just a small family of puppets!” Then, all the puppets were put in the box. During that phase, different events occurred with a potential impact on the number of puppets; they are specific to each experiment and will be described below. After this short delay, the experimenter and the child proceeded to wake up the puppets and put them back on the tree. No attempt was made to leave the same branch empty as in the starting configuration. The experimenter helped put the first puppets on the tree, leaving only two branches of the tree empty. She then handed the box to the child asking him/her to find “the rest”. Crucially, at that point, whatever the total number of puppets, there was only one puppet in the box (on trials with more puppets, the experimenter hid the last puppet in her hand), and this puppet was placed in the box such that it should be easy to find.

Once given the box, the child reached and found this puppet. The crucial measurement started when the puppet was placed on the tree: the child was given an 8-s time window during which searching in the box was recorded. During the searching window, the experimenter smiled and looked straight at the child, and intervened only if the child attempted to remove puppets from the tree. After 8 s, the experimenter asked the child a closing question (“Now do we have all the puppets?”) and then provided feedback. For the trials with one fewer puppets than branches, she said, “Yes we do! It is a small family of puppets”; for the other trials, she looked puzzled and reached in the box, sneaking the last puppet back into the box. The child was then invited to go and reach for the last puppet him/herself.

After they had participated in four experimental trials, children were given a short version of the give-N task. This task was intended to ascertain whether the children had words for exact integers (i.e., whether they were CP-knowers), rather than determine their exact knowledge level for small numbers. Children were presented with 15 rubber fish and a bowl (the “pond”). They were first asked

to put 3 fish in the pond. Depending on their success, in the next trial they were asked for  $N + 1$  fish, or for  $N - 1$  fish. If the children failed to give 3, then 2, then 1 fish (generally by compulsively putting all 15 fish in the bowl whatever the request), the method was changed, asking the child to put the fish in the hands of the experimenter, starting from a 1 fish request. Children were classified as subset-knowers once they failed at two requests for a number  $N$  (bowl or hands methods, whichever yielded better performance), even if they succeeded at numbers smaller than  $N$ . Children were classified as CP-knowers if they successfully gave 3, 4, and 5 fish.<sup>1</sup>

#### 2.4. Data coding

The data were video-recorded for later coding. Trials were excluded from the analyses if any of the following applied: (1) the child showed excessive distraction and did not comply with the trial script; (2) the child searched for the puppet that preceded the window where search was recorded for more than 10 s (we reasoned that if the child had to search for a long time for the  $N - 1$ th puppet, that would give him/her some indication that there was no  $N$ th puppet remaining in the box); or (3) the experimenter or the caregiver asked the closing question (“Now do we have all the puppets?”) before the end of the searching window. Two coders watched the video and used a custom-made python program in order to measure the searching time for the  $N - 1$ th puppet, the time the child searched for a  $N$ th puppet (within the 8-s time window starting when the  $N - 1$ th puppet was placed on the tree), and the time of occurrence of the question closing the trial, with respect to this searching window. Twenty-eight of the 324 trials were excluded from analyses, for experimenter error (4), excessive distraction (9), searching for the  $N - 1$ th puppet for more than 10 s (9 trials + 1 other trial where searching time for the  $N - 1$ th puppet could not be assessed), unclear searching behavior (2), or closing question asked too early (3). Because analyses were meaningful only if a child contributed data both in a trial where the box was expected to be empty and in a trial where the box was expected to contain one puppet, this resulted in the exclusion of 0–9 children from the analyses in each experiment.

When searching time was measured, the tape was played at slow speed (1/6), and each coder recorded searching by pressing keys on separate gamepads. The following behaviors were included in the searching time: (1) reaching inside the box (from the moment the child’s hand entered the box to the moment it exited), (2) looking inside the box (from the moment the child’s gaze was aligned with the opening of the box to the moment the child looked away), and (3) shaking the box to listen for noise (from the moment the child picked up the box to the moment when the shaking ended or the box was returned to the table). One of the coders was the experimenter, who advanced the tape at appropriate places. The other coder was blind to the condition and to the hypotheses. The program recorded agreement between coders by sampling their judgment (search/no search) every 30 ms. If the agreement was under 90% (16/279 trials), a second measurement was attempted, and the most convergent measurement was kept for analyses. For the final sample of 279 trials, the average agreement between the two coders was 98.8% (97.4% if considering only the trials with non-zero searching time). Analyses were conducted on the mean of the searching times measured by the two coders.

#### 2.5. Analyses

Results were analyzed using ANOVAs with one between- or within- subject factor for Condition (if appropriate), and one within-subject factor for Outcome (box expected empty vs. box expected to contain one puppet). Data within condition were analyzed with simple ANOVAs with one factor for Outcome. Preliminary analyses ensured that Gender, Order of presentation of outcomes (starting with a trial where the box was expected empty vs. was expected to contain one puppet), and trial Pair did not interact with Outcome in each experiment ( $ps > .05$ ).

<sup>1</sup> After the give- $N$  task, the children were prompted to count all the fish placed in a line on the table, in order to record their ability to recite the counting list. However, owing to the duration of the two first tasks, many children were tired at that point and refused to follow the experimenter’s instructions (caregivers nonetheless affirmed that the children could recite the first few count words). Therefore, the counting data were not considered in the analyses.

### 3. Experiment 1: Subset-knowers' use of one–one correspondence to reconstruct large sets over motion and occlusion

Experiment 1 tested whether subset-knowers could use one-to-one correspondence cues to reconstruct the exact number of objects in sets of 5 or 6 identical puppets, placed on a tree with 6 branches. In this basic situation, puppets were placed in an opaque box, and then returned to the tree after a short delay. After placing 5 puppets on the tree, children's searching time for a 6th puppet was compared across trials with sets of 5 and 6 puppets: if children could distinguish between these sets, they should search longer when the set consisted of 6 puppets.

All children were also tested on their ability to discriminate sets of 5 vs. 6 puppets in a second condition, where the branches of the tree did not provide additional information. This test was the same as the main experiment, except that the puppets were placed on a tree with 11 rather than 6 branches: thus, the number of empty branches when the puppets were placed on the tree was also 5 or 6. If the children were using the branches to reconstruct the exact number of puppets in the main experiment, their performance should drop in this second condition.

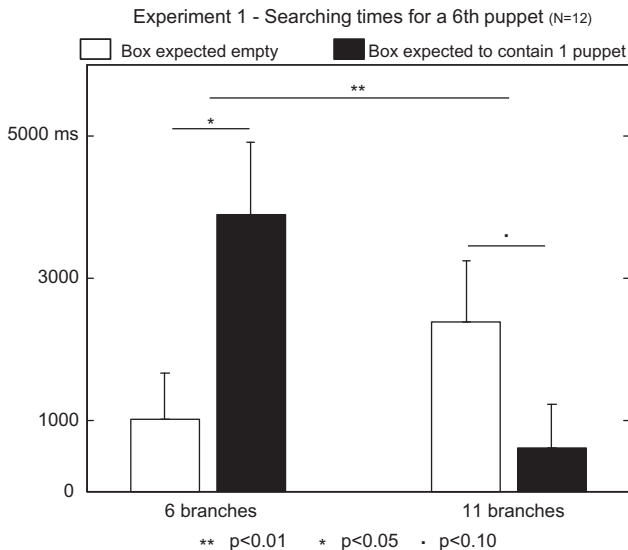
#### 3.1. Method

##### 3.1.1. Participants

The final sample of children consisted of 12 subset-knowers (8 female, mean age 34.14 months, 32:06–35:18).

##### 3.1.2. Procedure

Following the training procedure (see general methods), each child was given four experimental trials: two trials with a 6-branch tree, followed by two trials with an 11-branch tree. Trials started with 5 or 6 identical puppets placed on the tree. After the puppets were placed in the box, the box was shaken lightly while the experimenter told a brief story about the puppets sleeping. Half the children were tested with 5 puppets first, and half with 6 puppets first. Trials with 5 and 6 puppets were



**Fig. 2.** Searching times for Experiment 1. In Experiment 1, the set of puppets was simply returned to the tree after a short delay. The puppets were either placed on a tree with 6 branches (informative one-to-one correspondence cues) or on a tree with 11 branches (non-informative one-to-one correspondence cues). Error bars represent the standard error of the mean.

given in reverse order in the two parts of the experiment: for example, if a child received a trial with 5 puppets followed by a trial with 6 puppets in the 6-branch condition, he/she was first tested in the 11-branch condition with 6 puppets, then with 5 puppets.

### 3.2. Results

Fig. 2 presents the findings from this experiment. When the tree had six branches, the children were able to make an exact discrimination between sets of 5 and 6 puppets: they spent more time searching for a 6th puppet when the set really contained 6 puppets than when it contained 5 puppets,  $F(1, 11) = 5.0$ ,  $p = .047$ ,  $\eta_p^2 = .31$ . In contrast, when the branches were too numerous to support tracking of the set, searching was not significantly different for trials starting with 5 or 6 puppets,<sup>2,3</sup>  $F(1, 10) = 3.4$ ,  $p = .095$ ,  $\eta_p^2 = .25$ . The difference in behavior yielded a significant interaction between Set Size (5 or 6 puppets) and Condition (6 branches or 11 branches),  $F(1, 10) = 11.7$ ,  $p = .0065$ ,  $\eta_p^2 = .54$ .

### 3.3. Discussion

When the branches could be used to support the discrimination of sets of 5 vs. 6 puppets, children searched longer in the box when the last puppet was missing than when all puppets had already been retrieved. In contrast, they failed to solve the task when the correspondence between branches and puppets did not provide any useful information. Because all children were screened for knowledge of large number words using the diagnostic give-N task, and only those who failed this test were included in the experiment, the findings of Experiment 1 provide evidence that children can take advantage of one-to-one correspondence cues to make exact discriminations between large numbers of objects, before they learn symbols for large exact numbers.

These findings raise the question of whether children can make a further inference about one-to-one mappings: that such mappings are disrupted by the addition or the subtraction of one object. Experiment 2 addressed this question. Furthermore, we sought to obtain more data on the 11-branch condition, where branches were too numerous to support discrimination of 5 vs. 6 puppets; these new data would increase our statistical power and enable us to test whether subset-knowers could ever succeed in reconstructing large sets of objects, even without support from one-to-one correspondence cues. The full set of 11-branch results will be presented as Experiment 5, after the results of the experiments presenting informative one-to-one correspondence cues.

## 4. Experiment 2: Subset-knowers' use of one-to-one correspondence to reconstruct large sets of objects over addition or subtraction

Experiment 2 used the method of Experiment 1 to investigate whether subset-knowers could use one-to-one correspondence cues to reconstruct the exact number of objects in a set, after an addition or a subtraction of one item. As in Experiment 1, children first viewed sets of 5 or 6 puppets arranged on a tree with 6 branches. While the puppets were in the box, an event occurred that resulted in the addition or subtraction of either one puppet or one branch. If children could successfully take into account these additions and subtractions, they should search longer for sets containing 6 puppets at the end of the transformation event. If instead children disregarded the effects of the additions and subtractions, they should search longer on trials starting with as many puppets as branches, as in Experiment 1. Finally, it was possible that children might be uncertain about the effects of the additions and subtractions, in which case they might search equally across trials.

<sup>2</sup> One of the children did not contribute data in both 11-branch trials and was excluded from this analysis.

<sup>3</sup> The results of the 11-branch condition will be analyzed in further detail in Experiment 5. In the full data set, as here, we observed an unexpected reversal of searching times. Analyses performed on the full data set of Experiment 5 shed light on the origins of this effect (see Experiment 5 Results and discussion).

#### 4.1. Method

##### 4.1.1. Participants

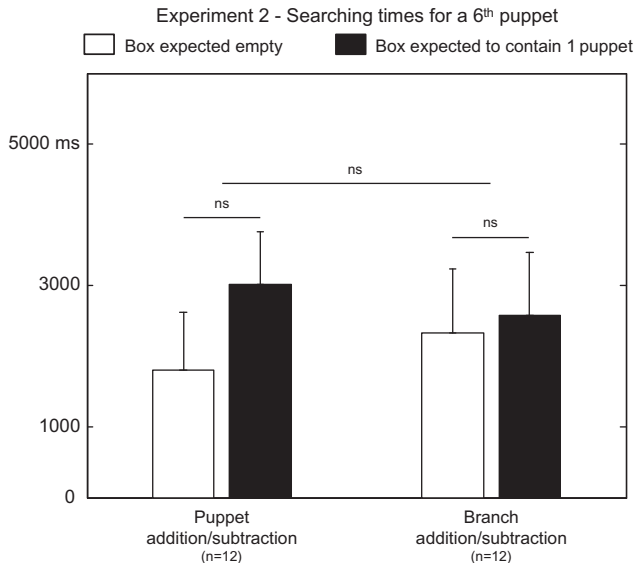
Participants were 24 subset-knowers (8 female, mean age 34.15 months, 32:15–35:26).

##### 4.1.2. Displays and procedure

All training and testing was as in Experiment 1, except that in the experimental trials, an additional event happened while the puppets were in the box. For one group of children, additions and subtractions were applied to the puppets (puppet addition/subtraction condition); for another group of children, additions and subtractions were applied to the branches (branch addition/subtraction condition). In each condition, children received two trials, one resulting in a group of 5 puppets, and one resulting in a group of 6 puppets, both to be placed on a tree with 6 branches (so we could compare searching across sets of 5 and 6 puppets, just as in Experiment 1).

For the puppet addition/subtraction condition, the outcome-6 trial started with a group of 5 puppets placed on a tree with 6 branches. Then, while the puppets were in the box, the experimenter took an extra puppet out her sleeve, and put it in the box, narrating, “Look at that, here is another puppet coming!”. The outcome-5 trial started with a group of 6 puppets, one per branch on the tree. After all the puppets were placed in the box, the experimenter reached in the box and removed one puppet, showed it to the child, and put it in a bag on the floor, narrating, “He does not want to sleep; he is going to the jungle”.

For the branch addition/subtraction condition, new trees were crafted such that one branch could be either added or removed (beginning with 5 or 7 branches and ending with 6 branches). The outcome-5 trial started with a tree with 5 branches (no empty branch). Then, while the puppets were in the box, the experimenter added a new branch to the tree, narrating, “That night, there is a big storm with lots of wind, a new branch is coming!” The rest of the trial unfolded as before with the tree now having 6 branches. The outcome-6 trial started with 6 puppets placed on a tree with 7 branches (one empty branch). Again, while the puppets were in the box, the experimenter described a storm in which one of the branches flew away, thus resulting in 6 puppets to be placed on a tree with 6 branches.



**Fig. 3.** Searching times for Experiment 2. In the puppet addition/subtraction condition, one puppet was either added to or subtracted from the set while the puppets were hidden in the box. In the branch addition/subtraction condition, one branch was either added to or subtracted from the tree while the puppets were hidden in the box. Error bars represent the standard error of the mean.

Following the two addition/subtraction trials, children were again given two trials in the 11-branch condition, as in Experiment 1. All the data of the 11-branch condition will be pooled together and analyzed as Experiment 5.

#### 4.2. Results

Fig. 3 presents the findings. In contrast to Experiment 1, children's searching time did not differ between the outcome-5 and the outcome-6 trials,  $F(1,22) < 1$ ,  $\eta_p^2 = .04$ . This was true of each condition tested separately:  $F(1,11) = 1.4$ ,  $p = .27$ ,  $\eta_p^2 = .11$  for the puppet addition/subtraction condition,  $F(1, 11) < 1$ ,  $\eta_p^2 < .01$  for the branch addition/subtraction condition, and no interaction was observed between Condition and Outcome size:  $F(1,22) < 1$ ,  $\eta_p^2 = .02$ . Children were not able to construct the correct one-to-one correspondence relation after the addition and subtraction events, whether the events were applied to a set that was invisible at the moment of the transformation (the puppets) or to a set that remained visible throughout the trial (the branches).

#### 4.3. Discussion

The findings of Experiment 2 provided no evidence that children appreciated how the operations of adding or subtracting should affect the one-to-one correspondence mapping between the puppets and the branches. There are several possible reasons for children's random search performance following the addition and subtraction events. One family of possibilities, to be evaluated in the next experiment, is that children failed to attend to, remember, or understand the transformations. Note that this failure could not have been absolute, since if the transformations were simply deleted from their memory, the children would have responded based on the size of the starting set as in Experiment 1, leading to below-chance performance in Experiment 2. Nevertheless, it is possible that children knew that something had happened, but did not understand exactly what had happened and how it impacted the set of puppets. Experiment 3 was undertaken to explore this possibility.

### 5. Experiment 3: Children's processing of addition and subtraction of one, applied to small sets of objects

Experiment 3 tested whether children could remember and process the transformations presented in Experiment 2 when the addition and subtraction events were performed on smaller sets, within the range of children's object-tracking abilities.

#### 5.1. Method

##### 5.1.1. Participants

Participants were 12 subset-knowers (7 female, mean age 33.94 month, 32:13–35:13).

##### 5.1.2. Displays

Displays were sets of 2 or 3 finger puppets, placed on a new tree that was constructed with only 3 branches. For the purpose of the branch addition/subtraction condition, additional trees were crafted to allow for the addition or removal of a branch (beginning with 2 or 4 branches and ending with 3 branches).

##### 5.1.3. Procedure

The children were first familiarized with the task using the same initial 3 trials as in Experiments 1 and 2. Following familiarization, children were given two trials in a puppet addition/subtraction condition, and two trials in a branch addition/subtraction condition, with order of presentation of condition and of trial outcome (2 puppets or 3 puppets) counterbalanced. The trials followed the procedure of Experiment 2, except with smaller sets of puppets and branches. For the puppet addition/subtraction condition, the outcome-3 trial started with 2 puppets on 3 branches, and the transformation event consisted in one puppet being taken from the sleeve of the experimenter and added to the



box. The outcome-2 trial started with a group of 3 puppets on 3 branches, and the transformation event showed one puppet being removed from the box and hidden in a bag on the floor. In the branch addition/subtraction condition, the outcome-2 trial started with 2 puppets on 2 branches, and one extra branch was added to the tree while the puppets were in the box. The outcome-3 trial started with 3 puppets on 4 branches, and one branch was removed from the tree while the puppets were in the box.

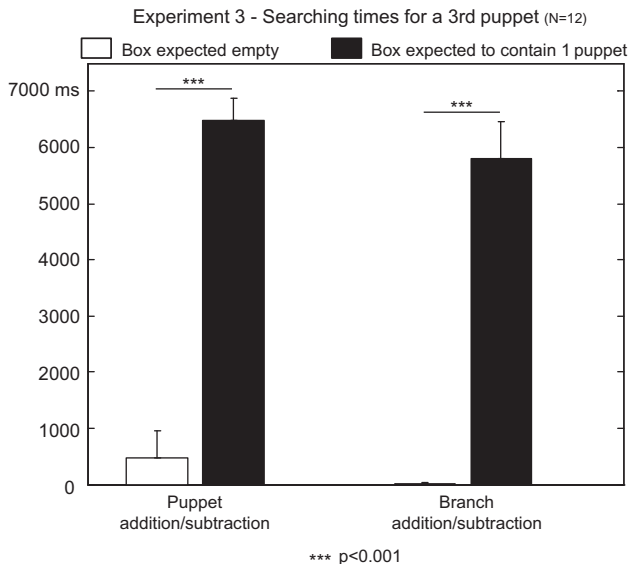
After each transformation event, the experimenter reached for the first puppet in the box and placed it on the tree. She then handed the box to the child. The searching time measurement started after the child had found the 2nd puppet and had placed it on the tree.

## 5.2. Results

Children solved this task easily (Fig. 4), showing a reliable effect of the Outcome size in the correct direction,  $F(1,10) = 307.7$ ,  $p < .001$ ,  $\eta_p^2 = .97$ . There was no effect of Condition,  $F(1,10) = 1.6$ ,  $p = .24$ ,  $\eta_p^2 = .14$ , and no interaction of Condition and Outcome Size,  $F(1,9) < 1$ ,  $\eta_p^2 < .01$ . Additional ANOVAs confirmed that, in each condition, the children searched longer for the 3rd puppet in the trials in which the transformation resulted in 3 puppets (puppet addition/subtraction condition:  $F(1,9) = 101.1$ ,  $p < .001$ ,  $\eta_p^2 = .92$ ; branch addition/subtraction condition:  $F(1,11) = 78.6$ ,  $p < .001$ ,  $\eta_p^2 = .88$ ). Furthermore, performance was significantly better with the small numbers of Experiment 3 compared to the large numbers of Experiment 2 (interaction between Experiment and Outcome Size for the puppet addition/subtraction condition:  $F(1,20) = 13.5$ ,  $p = .0015$ ,  $\eta_p^2 = .40$ ; for the branch addition/subtraction condition:  $F(1,22) = 15.0$ ,  $p < .001$ ,  $\eta_p^2 = .40$ ).

## 5.3. Discussion

In the context of small numbers, children were able to remember and process addition and subtraction transformations adeptly. They were equally able to do so whether the transformation affected a visible or an invisible set (branches or puppets). This finding converges with a host of research



**Fig. 4.** Searching times for Experiment 3. The two test conditions corresponded to the puppet and branch addition/subtraction conditions of Experiment 2, but with small numbers of puppets and branches. Error bars represent the standard error of the mean.

showing that children are able to infer and correct surreptitious transformations in small sets of objects (Gelman, 1972b; Gelman & Gallistel, 1986).

The children's success in Experiment 3 provided evidence that they were able to remember and understand the transformation events, thus excluding memory of the transformation events and other limits to processing the transformations as the reason for the children's failure in Experiment 2. Three potential explanations for this failure remain. First, perhaps children were able to remember a transformation event while tracking a small set of objects, but remembering both a transformation and a one-to-one mapping between branches and puppets exceeded their memory capacity. Indeed, in contrast to the conditions presenting large sets of puppets, it is possible that children did not use the branches to succeed with small sets, given that the set sizes did not exceed their object-tracking limit. Second, perhaps children remembered both the starting configuration and the transformation, but failed to combine these pieces of information so as to update their expectations for the final mapping between puppets and branches. In all the transformations used so far (additions and subtractions), the end configuration was different from the starting configuration, hence the need to update the mapping. Third, perhaps children of this age do not have a full understanding of whether transformations affect one-to-one mappings between sets; in other words, maybe children fail to recognize that relations established by one-to-one pairings follow the principle of *Addition/Subtraction*. Under this hypothesis, children in Experiment 2 were unsure whether the transformation events affected the one-to-one correspondence mapping between the branches and puppets, and thus they stopped attending to this mapping altogether. Experiment 4 was undertaken to distinguish these potential explanations.

## **6. Experiment 4: Children's use of one-to-one correspondence cues to reconstruct large sets through transformation events preserving number**

In Experiment 4, children were tested again with large numbers, but with transformations that did not affect one-to-one correspondence mappings, therefore removing the burden of having to update this mapping. As in Experiment 2, the transformations involved removing or adding one puppet to a box containing either 5 or 6 puppets. Two types of events were presented to the children. In the identity condition, one puppet first exited the box and then returned to the box after a short delay. At the end of the trial, the final set was thus composed of exactly the same individuals as at the start of the trial. The substitution condition differed in that the puppet returning to the box was a different individual from the puppet that left the box: This event thus preserved the number of elements in the set but not the identity of all its individual members.

If children were not able to combine information about one-to-one mappings with information about transformation events, for example by failing to remember both pieces of information at the same time, then they should fail to distinguish between the events involving 5 vs. 6 objects in either condition. If children interpreted one-to-one correspondence as establishing numerical equivalence (i.e., if they realized that additions and subtractions affect one-to-one mappings and that substitutions do not) but failed to compute the updated one-to-one mapping in the addition/subtraction conditions of Experiment 2, then they should succeed in both the identity and the substitution conditions. Finally, if children could use one-to-one mappings to establish a correspondence relation among specific objects, but not to establish numerical equivalence, then they should succeed in the identity condition but fail in the substitution condition.

### *6.1. Method*

#### *6.1.1. Participants*

Participants were 24 subset-knowers (16 female, mean age 34.04, 32:11–35:22).

#### *6.1.2. Displays*

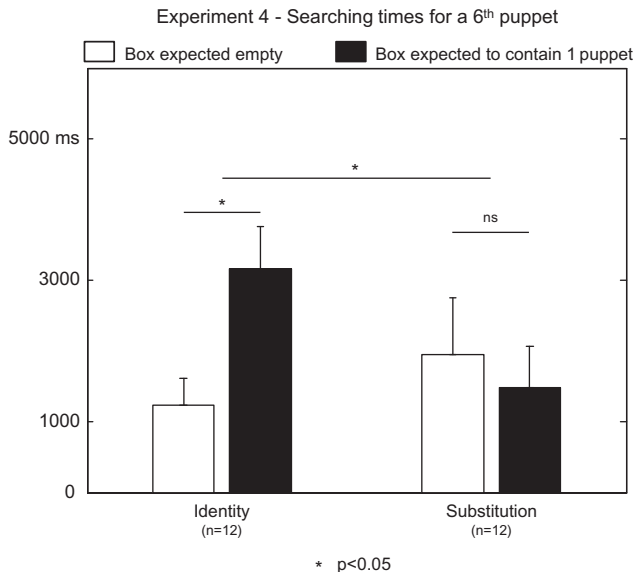
Displays were the same as in Experiments 1 and 2. A 6-branch tree was used, with sets of 5 or 6 puppets.

### 6.1.3. Procedure

Children received 4 experimental trials: two trials with sets of 5 puppets, and two trials with sets of 6 puppets, presented in a semi-alternating order as in past experiments. In both the identity and substitution conditions, the transformation event started with a puppet taken out of the box. In the identity condition, this puppet was returned to the box after narrating a cover story (“He is going to get a snack”). The cover story varied between the first and second pair of trials in an effort to maintain interest. The events in the substitution condition resulted in the substitution of one puppet by another identical puppet. Again, two story lines were used for the first and second trial pairs. In the first pair of trials, the substitution was enacted as a subtraction followed by an addition. The experimenter first took a puppet out of the box and placed it in a bag on the floor, narrating, “He does not want to sleep; he is going to the jungle”. Then, she took another puppet out of her sleeve and placed it in the box with the other puppets. For the second pair, we used a story line that emphasized the substitution, by showing two puppets swapping location. In this story, first the experimenter took a puppet from the box and placed it on the top of the box, narrating, “He is calling a friend”. She then took a second puppet out of her sleeve and proceeded to exchange the location of the two puppets: the puppet from the sleeve went to the box, and the puppet from the box went to the sleeve. In both events, the substitution puppet was strictly identical to the original puppet.

### 6.2. Results

Fig. 5 presents the findings. Children’s performance differed across conditions, as indicated by a significant interaction between the factors of Condition (identity vs. substitution) and Set Size (5 or 6 puppets),  $F(1, 22) = 4.5$ ,  $p = .046$ ,  $\eta_p^2 = .17$ . As in Experiment 1, children tested in the identity condition searched longer for a 6th puppet when the set contained 6 puppets,  $F(1, 11) = 8.1$ ,  $p = .016$ ,  $\eta_p^2 = .42$ . Thus, they were able to reconstruct the exact number of puppets over an intervening event that involved the removal and return of one element of the set but preserved the identity of each element. In contrast, children did not modulate their searching time with set size in the substitution condition,  $F(1, 11) < 1$ ,  $\eta_p^2 = .04$ .



**Fig. 5.** Searching times for Experiment 4. In the identity condition, one puppet was subtracted from the box and then added back, resulting in no change in the set. In the substitution condition, one puppet was subtracted from the box and then a different, featurally identical puppet was added to the box. Error bars represent the standard error of the mean.

### 6.3. Discussion

The findings of Experiment 4 provide evidence that children are able to preserve a one-to-one correspondence relation over events in which an object is removed from and then returned to a set, an event that does not change either the set's cardinal value or the identity of any of its members. This result confirms and extends the findings of Experiment 1, by showing that children are able to remember a one-to-one mapping between a large number of branches and puppets while attending to an intervening event. Indeed, the events presented in the identity condition were neither shorter nor simpler than those in the addition/subtraction conditions from Experiment 2; thus, children's patterns of success and failure across conditions could not easily be related to the complexity of the intervening transformation.

In contrast, children failed to use one-to-one correspondence relations to reconstruct a large set after a substitution event in which one puppet of the set was replaced by another puppet. Importantly, the identity and substitution transformations were equivalent in terms of numerical operations: one puppet exited the box, and later an identical-looking puppet entered the box. The children were nonetheless affected by the identity or distinctness of the puppets exiting and re-entering the box, i.e., whether a single individual participated in both transformations. These results provide strong evidence that the children were not processing the events numerically (in which case the two conditions would have been equivalent), and instead were registering individual objects. In other words, children used one-to-one correspondence cues as cues to specific individuals, rather than as markers of numerical equality. Set transformations posed difficulties for children, even when those transformations brought about no change in a one-to-one correspondence mapping.

Because of their theoretical importance, we sought to probe the robustness of the findings of Experiment 4 with a larger sample, and therefore we conducted two additional experiments (see detailed procedures and results in the [Appendix](#)). In Experiment 4B, we presented the identity and substitution events to 32 subset-knowers (16 female, average age 33.96 months, 32:00–35:29) in a within-subject design. Here again, the children used the one-to-one correspondence cues to reconstruct the sets after the identity events, 2495 ms vs. 3997 ms,  $F(1, 26) = 5.6$ ,  $p = .026$ ,  $\eta_p^2 = .18$ , but not after the substitution events, 1723 ms vs. 2301 ms,  $F(1, 29) < 1$ ,  $\eta_p^2 = .021$ ; however, this time the interaction between Condition and Set Size did not reach significance,  $F(1, 24) = 1.4$ ,  $p = .25$ ,  $\eta_p^2 = .05$ . We then performed a third experiment (Experiment 4C), which also served to evaluate the impact of the training procedure on children's use of branches as cues. Twenty-four children (13 female, average age 33.98 months, 32:05–35:26) were tested in the same conditions as in Experiment 4, except that the last training trial, designed to attract children's attention to the branches, was omitted. This time, the children's longer search for a set of 6 vs. 5 puppets failed to reach significance in the identity condition, 1812 ms (5 puppets) vs. 2247 ms (6 puppets),  $F(1, 11) = .33$ ,  $p = .58$ ,  $\eta_p^2 = .029$ , while searching times for the two sets again were equivalent in the substitution condition, 1260 ms vs. 1270 ms,  $F(1, 11) = .04$ ,  $p = .85$ ,  $\eta_p^2 < .01$ . Again, there was no interaction between Condition and Set Size,  $F(1, 22) = .35$ ,  $p = .46$ ,  $\eta_p^2 = .015$ .

We next pooled all the data together ( $n = 80$ ) in a mixed-model analysis to probe the robustness of the findings and perform comparisons across experiments. This analysis accorded exactly with the original findings of Experiment 4: we obtained a main effect of Set Size,  $\chi^2(1) = 6.8$ ,  $p = .009$ , a main effect of Condition,  $\chi^2(1) = 8.1$ ,  $p = .004$ , and most crucially, an interaction between these two factors,  $\chi^2(1) = 4.5$ ,  $p = 0.034$ . None of these effects was significantly modulated by Experiment (this was also true when Experiments 4B and 4C were compared separately with Experiment 4: see [Appendix](#)). In summary, while the pooled analysis indicated that the differences observed across experiments were not statistically reliable, it provided further support for the conclusions derived from Experiment 4: children were able to use one-to-one correspondence mappings to reconstruct exact sets through identity events, but not through substitution events.

In the next experiment, we return to children's ability to reconstruct exact sets in the absence of transformations. We ask whether subset-knowers show any ability to track a large number of objects when one-to-one correspondence cues do not provide useful information.

## 7. Experiment 5: Subset-knowers' enumeration of large sets in the absence of informative one–one correspondence relations

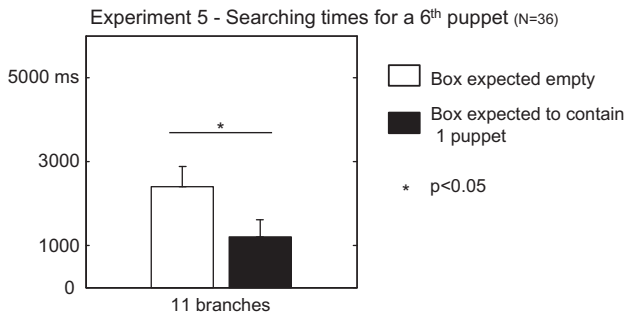
In Experiment 1, we showed that performance dropped with 11 branches compared to 6 branches, thus providing evidence that children detect and use the information provided by the one-to-one correspondence between branches and puppets. However, owing to the small sample size, the performance of this group alone did not reveal whether subset-knowers are at all able to reconstruct large exact numbers of objects, when one-to-one correspondence cues are not informative. We thus administered the 11-branch condition to the participants of Experiment 2 as well, in an effort to increase the sample. Here we present the data pooled for all participants in Experiments 1 and 2.

### 7.1. Method

The 11-branch condition was identical to Experiment 1 (no transformation), except that the sets of 5 or 6 puppets were now placed on a tree with 11 branches, thus making a difference of one puppet harder to detect. Children received two trials in the 11-branch condition (one with 5 puppets, one with 6 puppets), after completion of the two trials of Experiment 1 or 2. In total, 36 subset-knowers (16 female, mean age 34.08 months, 32:06–35:26) contributed data for both set sizes (5 and 6 puppets): 13 participants from Experiment 1, 13 participants from the puppet addition/subtraction condition in Experiment 2, and 10 participants from the branch addition/subtraction condition in Experiment 2.

### 7.2. Results and discussion

Fig. 6 presents children's performance in this experiment. There was no difference between the subgroups of children who had previously participated in different experiments or conditions ( $ps > .24$ ,  $\eta_p^2 < .09$  for the main effect and interaction involving Subgroup) so the data were pooled across these experiments and conditions. Children's performance was opposite in direction to the correct pattern: they searched longer in the trial in which no puppet should have remained in the box (5-puppet trial) than in the trial in which one puppet should have remained (6-puppet trial),  $F(1, 33) = 4.4$ ,  $p = .043$ ,  $\eta_p^2 = .12$ . This seemingly counterintuitive result appears to be an effect of the feedback received on the first trial: on the second trial, children tended to align their searches with this feedback. Hence, children tended to search less after a first trial with 5 puppets, in which no further search was warranted (3072 ms searching with 5 puppets followed by 887 ms searching with 6 puppets); in contrast, the searching time increased slightly after a first trial with 6 puppets, in which the feedback had shown one puppet to be missing (1467 ms searching with 6 puppets followed by 1874 ms searching with 5 puppets). This pattern resulted in an interaction between Set Size and Trial Order,  $F(1, 34) = 5.7$ ,  $p = .023$ ,  $\eta_p^2 = .14$ .



**Fig. 6.** Searching times for Experiment 5. The tree had 11 branches, 5 or 6 more than there were puppets, thus the branches did not convey any useful information to track the puppets. Error bars represent the standard error of the mean.

In summary, the data from the 11-branch condition provide no evidence that subset-knowers can succeed in tracking large exact numbers of objects when one-to-one correspondence cues are not informative. However, these data should be interpreted with caution, given that the 11-branch trials were always presented after children had participated in another experiment on a 6-branch tree, and also had received a familiarization trial to orient them to attend to the tree.

## 8. General discussion

In the present research, we tested whether children who do not yet possess symbols for large exact numbers (subset-knowers) are nonetheless able to give judgments pertaining to large exact quantities. To do so, the children were provided with one-to-one correspondence cues indexing the objects of a set: cues that made exact numerical differences accessible to perception. In conditions where the set to be reconstructed was comprised of the same individual items throughout the trial (no transformation in Experiment 1; the identity-preserving events in Experiment 4), the children were able to discriminate 5 from 6 puppets. The information conveyed by the one-to-one correspondence cues proved essential to the children's success, as their performance dropped when these cues were not informative (Experiment 5). Our findings therefore provide evidence that children understand at least some aspects of Hume's principle before they acquire symbols for exact numbers: they understand that one-to-one correspondence provides a measure of a set that is exact and stable in time, even through displacements and temporary occlusions.

However, as soon as a transformation affecting either the identity of the set to be reconstructed (the puppets) or the identity of the one-to-one correspondence cues (the branches) was applied (additions and subtractions in Experiment 2, substitutions in Experiment 4), our participants ceased to perform exact discriminations on large sets. In contrast, Experiment 3 provided evidence that children performed near ceiling when the same addition and subtraction events were applied to small sets, thus excluding memory for the transformation itself as the source of the children's difficulty. Furthermore, Experiment 4 presented a minimal contrast between two events that each resulted in no change in number: one event that did not affect the identity of the individual members of the set (one puppet exiting and re-entering the box) and one event that did (one puppet exiting the box and another, featurally identical puppet entering the box). Although the same puppet movements occurred through the opening of the box in these two conditions, children succeeded at reconstructing the sets in the former case and failed in the latter. Interestingly, children did not ignore the transformation altogether, for they did not expect the end set to stand in a similar one-to-one relation to the branches of the tree as the starting set. Rather, whenever the identity of the items in the set of puppets changed, the children appeared to give up on the one-to-one correspondence cues and switched to a generic strategy, searching until they felt the box was empty.

How can this pattern of success and failure be explained, and in particular, why did children succeed only when the identity of the puppets and the branches was preserved? Perhaps their performance simply reflected their understanding of our task: after all, in real life there are situations where identity, not number, is the relevant factor. When gathering your family in your house, for example, it is important to make sure that your own children are there: replacing them with the neighbor's will not do. Despite the fact that the experimenter was calling the set of puppets a 'family', several pieces of evidence indicate that children did not interpret the goal of the present task as being restricted to the individuals presented on the tree at the start of the trial. Crucially, when tested with small sets, they readily placed all puppets on the tree, even when one of them was a newcomer. Furthermore, with large sets they failed to solve the task following the addition or subtraction of a branch, despite the fact that the family of puppets did not change in this condition. Thus, the pattern of findings obtained with large sets evidently reflects limitations to children's processing of these sets, rather than their understanding of the task.

Perhaps children's performance with large sets was constrained by limitations of processing resources, such as limitations in working memory<sup>4</sup>: the children may have failed to remember all the rel-

<sup>4</sup> In all the studies, we embedded the transformation events in narratives, in the hope that these narratives would make the events both more appealing and more memorable to children. However, it is an open question whether the narratives enhanced children's interest in or memory for the events.



evant pieces of information, or to process this information appropriately. Because children succeeded with the identity-preserving events and in the absence of any transformation, we know that they could remember one-to-one relations between branches and puppets and reproduce such a relation at the end of a trial. Furthermore, because they succeeded at tracking additions and subtractions with small sets, we know that they could remember and process set transformation events. However, it is possible that the joint requirements of remembering both a one-to-one mapping and a transformation exceeded the limits on children's memory and attention. Alternatively, even if children could remember all the relevant information, they might have failed to combine these two pieces of information to predict the final mapping between branches or puppets.

Crucially, our task was designed so that there were strategies available for working around any limitations in children's processing resources. First, in the substitution events, children could have succeeded by focusing on the initial state of one-to-one correspondence and discarding the transformation as having no effect. Children were likely to discover this strategy, however, only if they understood that a subtraction of one is reversed by an addition of one. Second, in all conditions, children could have succeeded by tracking the set of branches that were not paired with a puppet, rather than the set of puppets itself. Because there was never more than one unpaired branch, this set was always fully within the range of children's object-tracking capacities. To succeed with this latter strategy, however, children needed (1) to understand that tracking branches would yield the same information as tracking puppets, and (2) to represent transformation events in terms of their impact on the set of unpaired branches. For example, an addition of one puppet corresponded to one fewer unpaired branch, a subtraction of one puppet corresponded to one more unpaired branch, and so on. Perhaps, this mental operation was not available to children, and thus limited their use of strategies based on tracking branches. Although this difficulty may explain children's failure with transformations involving puppets (addition/subtraction or substitution), it fails to account for children's failure at the branch addition/subtraction condition, where the impact of the events on the set of unpaired branches was easily identifiable. This last finding thus leads us to favor the alternative explanation, i.e., that children failed to realize that the task could be solved not only by tracking the puppets, but also by computing how many branches did not have a matching puppet – a limitation of their understanding of one-to-one correspondence relations.

Children's format of representation for one-to-one mappings may have been such that they could not easily track the set of unpaired branches through transformations. One-to-one correspondence relations may be represented either via individual pairings (as in "each branch has a puppet") or at the level of the whole set. In the first case, to represent the puppets in relation to the branches, children could use their resources for parallel object tracking, with the branches serving as a support to expand the capacities of this system. A relation with one fewer puppets than branches could be represented using two slots in memory, one for the generic relation ("each branch has a puppet") and one for the deviant branch. This format of representation, however, should be easy to update following the addition or subtraction of a branch, which leads us to favor an alternative hypothesis. Instead of representing the relation at the level of individuals, children may have encoded the mapping between branches and puppets as a visual configuration, which, sometimes (e.g., when the identity of the set was preserved), they tried to reproduce as they were taking the puppets out of the box. In line with our results, such an ensemble-based representation of the relation between puppets and branches would not easily enable children to compute the impact of one-item transformations, be they transformations of puppets or of branches. This second possibility thus appears more likely, but further research is needed to distinguish these alternatives.

Whatever the reasons for children's failures, the present pattern of results indicates that one-to-one correspondence does not specify exact numerical equivalence for children of this age. Children know that transformations might affect how sets can be measured by one-to-one correspondence, but they are unable to predict which transformations do or do not affect this measure. Prior to the mastery of number words and counting, children thus do not recognize that one-to-one correspondence pairings instantiate all of the properties of the relation of exact numerical equality: more specifically, they recognize that one-to-one correspondence pairings are stable as long as the sets remains identical (*the Identity principle*) but not how these pairings are affected by additions, subtractions, or substitutions applied to one set (*the Addition/Subtraction and Substitution principles*). Our findings thus

stand in contrast both to the thesis that children who have not mastered counting can represent only purely approximate ensembles of objects, and to the thesis that such children represent exact number. On the one hand, children's understanding goes beyond approximate equality, because when they track a set that remains identical, they are sensitive to its exact number of elements. On the other hand, their understanding does not entail all aspects of the mathematical definition of exact number. To acquire a full concept of numerical equality, children may later enrich this initially restricted concept of identity.

### 8.1. *Young children's interpretation of one-to-one correspondence: the importance of transformation events*

Our findings replicate and extend previous reports that young children sometimes use one-to-one correspondence as a successful strategy for producing or evaluating sets of objects. For example, subset-knowers can judge whether two sets aligned in visual correspondence are “the same” or not (Sarnecka & Gelman, 2004). Young children also use one-to-one correspondence spontaneously when sharing a set among several recipients (Mix, 2002). In Piaget's experiments, moreover, children use one-to-one correspondence to construct sets of the same number (Gréco & Morf, 1962; Piaget, 1965). Finally, set-reproduction tasks have been used to assess knowledge of exact quantities in populations of children and adults without access to exact numerical symbols (Butterworth, Reeve, Reynolds, & Lloyd, 2008; Everett & Madora, 2012; Flaherty & Senghas, 2011; Frank et al., 2008; Gordon, 2004; Spaepen, Coppola, Spelke, Carey, & Goldin-Meadow, 2011).

However, the use of one-to-one correspondence strategies in set-matching tasks cannot stand as definitive evidence for understanding exact equality, for two reasons. First, across different versions of set-reproduction tasks, marked differences in performance have been observed when the spatial distribution or the nature of the items to be matched were varied: participants generally showed high performance when the model and response sets were visually aligned, and much lower performance when these two sets were presented in different modalities or spatial configurations, or when one of the sets was hidden from view as the participants gave their responses (Frank et al., 2008; Gordon, 2004; Spaepen et al., 2011; see Frank et al., 2011, for a reproduction of these patterns of difficulty in a group of US adults under verbal interference). This variability across conditions raises questions for the interpretation of the results: Should we grant participants understanding that one-to-one correspondence entails exact equality, when they only use one-to-one correspondence for two sets that are visually aligned? Or should we only draw this conclusion when one-to-one correspondence is used systematically, for all kinds of displays?

Second, set-reproduction tasks can overestimate people's understanding of exact equality. If a person lacks the concept of exact numerical equality altogether and aims to construct a set approximately equal to a target set, using one-to-one correspondence would be a successful strategy to do so: the resulting set would indeed be approximately equal to the model set (in fact, unbeknownst to the set-maker it would even be better than approximately equal, if no mistake has been made). In line with this observation, Gréco & Morf (1962) noted that some young children switch between one-to-one correspondence and estimation strategies when trying to match the numerosity of an array, as if they did not understand that these two strategies give results of a different nature. Thus, children or adults who have not mastered counting may use one-to-one correspondence as a strategy to achieve an approximate numerical match, without trying to reproduce the numerosity of the target exactly.

Although set reproduction is not in itself a strong test of one's concept of number, eliciting judgments on the impact of set transformations on one-to-one correspondence relations, as in our task, provides more definitive evidence (see also Izard et al., 2008; Lipton & Spelke, 2006; Spaepen et al., 2011). By eliciting judgments on one-item transformations, we were able to characterize the properties children attribute to one-to-one correspondence mappings, and contrast their conception of one-to-one correspondence with true numerical equality. We found that young children's interpretation of one-to-one correspondence encompasses only a subpart of the properties of numerical equality: an understanding that falls short of possessing a concept of exact number. Further research should employ the same type of tasks with other populations, in particular populations without symbols

for exact numbers, to evaluate the role these symbols play in the emergence of a concept of exact numerical equality.

## 8.2. Consequences of our findings for the debate on children's interpretation of number words

As we noted in the introduction, past research investigating whether subset-knowers construe number words as referring to exact quantities has yielded mixed results (Brooks et al., 2012; Condry & Spelke, 2008; Sarnecka & Gelman, 2004). More specifically, out of the four tasks reported in the literature, children failed to interpret number words as referring to exact quantities in three cases.

In a first task, subset-knowers were presented with two sets of toys, one labeled with a number word beyond their knowledge range (e.g., “five”), and the other unlabeled. They were then asked to point to a set designated either by this original number word (“five”) or by a different number word (e.g., “ten”). In this task, children correctly pointed to the set the experimenter had labeled when they heard the same number word, and to the other set when they heard the different number word—as long as no transformation was performed on either set. Whenever the experimenter applied a transformation to the labeled set (rearrangement, addition, or subtraction) before asking the same question, the children responded at chance: they did not consistently apply the original number word to a set that had been rearranged, and they did not consistently apply a different number word to a set that had been transformed by addition or subtraction (Brooks et al., 2012; Condry & Spelke, 2008). Thus, in this first task, children did not apply number words to exact quantities.

One may object that this first task was overly complex, but subset-knowers have been found to perform as poorly in a seemingly simpler task (Sarnecka & Gelman, 2004; Sarnecka & Wright, 2013). There, children were presented with two sets aligned in one-to-one correspondence, thus highlighting any difference between them. Across trials, sets either were exactly equal in number or differed by one item. The experimenter labeled one of the sets with a number word and asked the child about the second set, giving a choice between the same and a different number word. Although children were able to state whether the two sets were the same or not in a pretest question, they did not use this similarity to choose between the two proposed number words.

In a different task (Brooks et al., 2012; Sarnecka & Gelman, 2004), children had to judge whether a number word continued or ceased to apply to a single set of objects that were placed in an opaque box and transformed through addition, subtraction, or rearrangement (shaking the box). In contrast to the above findings, subset-knowers reliably chose the original number word after the shaking event, and they chose the alternative number word after the addition or subtraction transformation, this time behaving as if they interpreted number words as precise.

Finally, in a fourth study, subset-knowers were again tested with a single set of objects that was labeled with a number word and then transformed. This study differed from the previous one in three respects: First, instead of adding or subtracting just one object, the number of objects was doubled or halved; second, this time the sets remained fully visible throughout the transformation; and third, children were asked whether the original number label, or a different label, now applied to the set, rather than given a choice between two labels (Condry & Spelke, 2008). This time, children's performance dropped to chance with the addition and subtraction events (although they answered correctly when no transformation was applied), contrasting with the results of the previous transformation task.

Several explanations have been proposed for this pattern of results. First, children's failure at the last task suggests that keeping the sets visible may have a negative impact on their performance. When sets are visible, children may be drawn to rely on perception, which is approximate, and thus to generalize number words beyond exact numerical quantities. However, this explanation seems unlikely, because (1) Condry and Spelke's (2008) visible single-set task induced major changes in numerosity (doubling and halving), easily detectable by children, and (2) children failed at Sarnecka and Gelman's (2004) one-to-one comparison task, where the conditions of presentation highlighted any difference across sets.

Second, it is possible that tasks involving two sets are simply overwhelming for children, single-set tasks thus being a better indicator of children's semantic competence (Sarnecka & Gelman, 2004). However, Condry and Spelke (2008) showed that children sometimes succeed in two-set tasks, since

participants solved the task with high accuracy when no transformation was applied to the sets, and they also showed that participants sometimes failed in single-set tasks.

Third, counter to the previous explanation, Brooks et al. (2012) argued that children succeeded at Sarnecka and Gelman's (2004) single-set transformation task without extensive knowledge of the semantics of the number words. According to their argument, to succeed at the task children only need to know that a change in quantity is necessary to warrant a change of number word: therefore, children know to conserve the initial label after a shaking event. For addition and subtraction transformations, however, they find the right answer only by applying pragmatic inferences: If a child is given a choice between a label he/she heard earlier in the trial and a new label, Brooks et al. argue, given the assumption that the adult asking the question is knowledgeable, the child would infer that the new label provided is relevant. Pragmatic inferences, in contrast, provide no ground to find the correct answer in Condry and Spelke's (2008) two-sets task. To support this view, Brooks et al. adapted Condry and Spelke's (2008) two-set task and Sarnecka and Gelman's (2004) single-set transformation task using novel words and objects, and obtained the same pattern of success and failure across these two tasks, where children were asked to choose between two labels (as in Sarnecka and Gelman's single-set task) or between two objects (as in Condry and Spelke's two-sets task).

This last explanation holds promise to explain the whole set of results, with one adjustment: Given the contrast between children's reasoning about identity and substitution events in Experiment 4, children may not think that a change in number words requires a change in *quantity* but rather a change of *set identity*. According to this interpretation, children would be confident that a number word continues to apply to a set as long as this set remains composed of all and only the same individuals. In contrast, just like in the case of addition or subtraction transformations, they would make no specific prediction as to whether this number word or another number word applies, if one or more individual members of the set are replaced by other individuals – unless the pragmatics of the task leads them to the correct answer.

This explanation in terms of set identity predicts children's failure at the one-to-one comparison task, which was left unexplained in Brooks et al.'s (2012) account. Indeed, in both the one-to-one comparison task and the single-set transformation task, children must choose between a previously-heard label and a new label, thus in terms of pragmatics the two tasks are equivalent. In terms of quantities involved, the two tasks are equivalent too. Therefore, if children reason in terms of quantity, they should succeed in the comparison task when the two sets are equal in number, just as they succeed in the single-set task when no transformation is applied. If however children reason in terms of set identity, then in the one-to-one comparison task there is no reason why information about one set should help them solve a question about another set. To get a better understanding of this interpretation, think of first names, which are defined in terms of identity. If a set is called “five” and is put in exact one-to-one correspondence with another set, we predict that children are undecided as to whether this second set should be called “five” like the other set. Nevertheless, children should know that if the members of a set called “five” remain in the set, and no new item is added, then the set is still called “five”.<sup>5</sup>

Interpreting children's usage of the number words in terms of set identity makes an important prediction. In the published versions of the single-set transformation task (Brooks et al., 2012; Sarnecka & Gelman, 2004), the transformation leaving numerosity constant left the identity of the set unchanged as well. Under our interpretation, subset-knowers should not choose to conserve the initial number word for an identity-changing substitution transformation, even though the cardinal value of the set remains constant in this condition.

<sup>5</sup> Set identity fails to account for one of the findings cited above: the fact that, in Condry and Spelke's (2008) two-sets task, children failed to modulate their response when asked to choose a set corresponding to the original vs. a new number word, not only after addition and subtraction events, but also after mere rearrangements. Instead, in all cases, whether presented with the original or with a new number word, children tended to pick the set that the experimenter had manipulated. In this task, it is possible that the rearrangement event, which was executed by the experimenter purposefully (in contrast to the shaking events that have been employed in other tasks) called children's attention to the manipulated set or induced the children to consider that the experimenter's action ought to be relevant to her question.

### 8.3. Understanding numerical equality: going beyond identity

At 5 years of age, children have clearly overcome the limitations of their understanding of numerical equality, since they know how set transformations impact number words, even for number words that fall beyond their counting range, and even for substitution transformations that keep number constant while altering the identity of a set's members (Lipton & Spelke, 2006). Furthermore, this knowledge is not restricted to the use of numerals, as 5-year-old children also use visual one-to-one correspondence as a criterion to judge whether two sets are equal or not, and they can predict the effect of additions and subtractions depending on the starting relation (Cooper, 1984).

How do children progress from an initial understanding of set identity to the adult concept of numerosity? One possibility is that children first understand the principles of exact numerical equality as applied to small sets, through their object-tracking system, and later extend those principles to large sets (Klahr & Wallace, 1973). As far as understanding the impact of addition and subtraction transformations on numerical equality, this seems a likely possibility, given children's ability to predict the numerosity of small sets through addition and subtraction events. However, it remains to be shown that young children are able to handle substitution events with small numbers, since substitutions are necessarily more complex: they are formed of at least two simple events, one addition and one subtraction.

Alternatively, experience with numeric symbols may play a crucial role in the acquisition of exact numerical equality. As children become CP-knowers, they assign a meaning to number words that is defined in terms of the counting procedure. Although the impact of the transition to the CP-knower stage on children's concepts of number is debated (Davidson et al., 2012; Le Corre et al., 2006), all parties agree that, at a minimum, CP-knowers appreciate that to say that there are 'five frogs' means that if they count this set of frogs, they will end the count with the word 'five'. Thus, CP-knowers have access to a representation that has the properties of exact numbers, and in particular, implies a relation of exact numerical equality between sets. As a result, whenever they are able to apply counting, or perhaps even when they can simulate the application of counting, CP-knowers gain the ability to respond in accordance with a precise interpretation of number words. For example, contrary to subset-knowers, CP-knowers generalize number words correctly in face of two sets presented in visual one-to-one correspondence (Sarnecka & Gelman, 2004; Sarnecka & Wright, 2013), perhaps because this configuration enables them to predict how the results of counts would compare across these two sets.

In other tasks where counting is not permitted, young CP-knowers sometimes revert to the same errors as subset-knowers (Davidson et al., 2012; Sarnecka & Carey, 2008). Nevertheless, it is possible that, after the children have become CP-knowers, the counting procedure serves to scaffold the development of a concept of exact numerical equality between sets by providing children with a mental model from which they derive the properties of exact numbers. In line with this proposal, words seem to have a special status with respect to one-to-one correspondence relations for children, as preschoolers rely on one-to-one correspondence much earlier when it entails a correspondence between words and some objects, than when two different sets of objects are put in correspondence with respect to one another (Gréco & Morf, 1962; Muldoon, Lewis, & Freeman, 2009). A similar observation was made with a group of adult homesigners from Nicaragua (Spaepen et al., 2011): In a series of set-reproduction tasks, homesigners used one-to-one correspondence strategies only rarely, and when they did so, they used it by mapping their fingers (the constituents of their number signs) to objects, never by mapping two sets directly onto each other, a seemingly simpler strategy. Understanding how words (or, in the case of homesigners, fingers) stand in one-to-one correspondence with objects while counting may be the first step that leads to a more general understanding of one-to-one correspondence relations, and in particular of how one-to-one correspondence warrants exact numerical equality.

## 9. Conclusion

Our findings shed light both on the extent and the limits of children's numerical knowledge, before they master the meanings of all the number words they use in counting. Children who have not mastered the exact numerical meanings of "five" and "six" are able to use one-to-one correspondence cues to reconstruct a set of exactly five or six objects, even when the sets are moved around, rearranged in

space, and kept out of view for some time, and even if one individual is first subtracted and then added back to the set, as long as the identity of the items forming the sets is not modified. However, children do not know how set transformations that change the individual members affect the way sets can be measured by one-to-one correspondence. Hence, before children acquire symbols for exact number, one-to-one correspondence defines a relation of identity between sets: a relation that is not limited to approximate numerical equality but falls short of exact numerical equality. Furthermore, children do not understand how one-to-one mappings interact with the addition of one, i.e. the successor function. At 3 years of age, the child's state of knowledge for number thus corresponds to the initial stage of Russell–Frege's formal definition of cardinal integers: they have a relation of set identity, but yet have not figured out how this notion interacts with basic operations, and how the numbers can be ordered in a list structured by a successor function.

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## Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at <http://dx.doi.org/10.1016/j.cogpsych.2014.01.004>.

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